FIELD DEPENDENCE OF THE GROUND STATE IN FINITE AMORPHOUS ISING SYSTEMS WITH FRUSTRATION

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Ground state properties of computer simulated disordered and frustrated finite two-dimensional Ising systems in an external magnetic field are analyzed in terms of the lowest states of a given magnetization and compared with an ordered system. Magnetization jumps for intermediate fields are interpreted.

Recently attention has focused on exact results for finite spin-glass models [1-7]. Using methods of discrete optimization theory the exact ground state of frustrated Ising $S = 1/2$ systems has been calculated up to $N = 484$ for $\pm J$ couplings on a square lattice [3-5] and up to $N = 60$ for two-dimensional amorphous systems with distance dependent short-range antiferromagnetic interactions [1,8,9]. Here we consider the same type of amorphous antiferromagnetically interacting random packings of hard disks (see ref. [1] for more details) and extend the calculations to the case of non-zero external fields.

The ground state energy is

$$E^0 = - \sum_{i < j} I_{ij} S_i^0 S_j^0 - H \sum_i S_i^0 = E_{M}^0 - MH, \quad (1)$$

where $S_i^0$ is the spin variable ($\pm 1$) of the $i$th spin in the ground state configuration. We divide (1) by $N$ and normalize to the “ideal energy per spin” $\epsilon_{id} = \sum I_{ij}/N$. With $\epsilon = E/N\epsilon_{id}$, $h = H/\epsilon_{id}$ and $m = M/N$ we obtain $\epsilon^0 = \epsilon_{M}^0 - mh$. In fig. 1b $\epsilon^0$ vs. $h$ is shown for amorphous systems up to $N = 40$ (AAFM). Because of the linear part in (1) it is obvious that $\epsilon^0(h)$ is determined by the energy of the respective lowest states of a given magnetization for zero field $\epsilon_{M}$, which are represented as intersection points of the straight lines in fig. 1 with the ordinate.

For comparison fig. 1a shows the situation in the corresponding unfrustrated ordered case (square lattice with nearest-neighbour antiferromagnetic interaction OAFM), where $\epsilon_{M}$ depends linearly on $M$, since $\epsilon_{M}$ and $\epsilon_{M+2}$ differ by single spin excitations. It follows $\epsilon_{M} = \epsilon_{0}^0 + (1 - \epsilon_{0}^0)m$ and all straight lines $\epsilon(h) = \epsilon_{M} - mh$ intersect at a critical field $h_c = 2$. Therefore the ground state (the lowest lying line) for all $h < h_c$ belongs to the antiferromagnetic order ($\epsilon^0(AFM) = -1$) and for all $h > h_c$ to the ferromagnetic one ($\epsilon^0(FM) = 1 - h$).

Frustration leads to an increase of the ground state energy in comparison with OAFM, which is marked by an arrow in fig. 1b. Moreover, it also influences the $M$

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Fig. 1. Field dependence of the ground state energy $\epsilon^0(h)$ for (a) ordered square antiferromagnet OAFM (thick line); (b) computer simulated amorphous short-range antiferromagnetic systems AAFM: $\bigcirc N = 10$, $\bullet N = 30$ (5 systems), $\times N = 40$; the lowest states of a given magnetization in zero field $\epsilon_M$ vs. $m$ (upper abscissa) is shown by broken lines.
dependence of the $\epsilon_M$ values the stronger the lower $\epsilon_M$ lies. For the AAFM our simulations suggest

$$\epsilon_M = \epsilon_0^0 + (1 - \epsilon_0^0) m^2 \pm \ldots$$

(2)

Again, the lowest of the straight lines $\epsilon(h) = \epsilon_M - mh$ determine the ground state energy for a given $h$. In consequence of the approximation (2) $\epsilon^0(h) \propto h^2$ and $m \propto h$ results.

Fig. 2 shows $\epsilon(m)$ with $h$ as parameter, where the minima of the curves belong to the ground state. The flat minima for intermediate fields in connection with fluctuations in $\epsilon_M$ due to disorder lead to metastability: States with different magnetization are nearly of the same energy. Consequently, changes of $h$ in this region cause jumps of magnetization (fig. 3), that means the magnetization process is a sort of "devil's staircases", cf. ref. [10].

Fig. 2. $\epsilon = \epsilon_M - mh$ vs. $m$ for one AAFM system ($N = 30$) and $h = 0 (\epsilon_M(m))$, $h_1 = 1.2$, $h_2 = 1.9$, $h_3 = 2.4$. The arrows mark the ground states. The $\times$ states do not occur as ground states for any $h$. The $\Delta m_j$'s denote magnetization jumps.

Fig. 3. Magnetization vs. field for the AAFM system ($N = 30$) of fig. 2.

References