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FIELD DEPENDENCE OF THE GROUND STATE IN FINITE AMORPHOUS ISING SYSTEMS WITH FRUSTRATION

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Ground state properties of computer simulated disordered and frustrated finite two-dimensional Ising systems in an external magnetic field are analyzed in terms of the lowest states of a given magnetization and compared with an ordered system. Magnetization jumps for intermediate fields are interpreted.

Recently attention has focused on exact results for finite spin-glass models [1–7]. Using methods of discrete optimization theory the exact ground state of frustrated Ising $S = 1/2$ systems has been calculated up to $N = 484$ for $\pm I$ couplings on a square lattice [3–5] and up to $N = 60$ for two-dimensional amorphous systems with distance dependent short-range antiferromagnetic interactions [1,8,9]. Here we consider the same type of amorphous antiferromagnetically interacting random packings of hard disks (see ref. [1] for more details) and extend the calculations to the case of non-zero external fields.

The ground state energy is

$$E^0 = -\sum_{i<j}^{N} I_{ij} S_i^0 S_j^0 - H \sum_{i}^{N} S_i^0 = E_M^0 - MH, \qquad (1)$$

where S_i^0 is the spin variable (± 1) of the ith spin in the ground state configuration. We divide (1) by N and normalize to the "ideal energy per spin" $\epsilon_{\mathrm{id}} = \sum |I_{ij}|/N$. With $\epsilon = E/N\epsilon_{\mathrm{id}}$, $h = H/\epsilon_{\mathrm{id}}$ and $m = M/N$ we obtain $\epsilon^0 = \epsilon_M^0 - mh$. In fig. 1b ϵ^0 vs. h is shown for amorphous systems up to $N = 40$ (AAFM). Because of the linear part in (1) it is obvious that $\epsilon^0(h)$ is determined by the energy of the respective lowest states of a given magnetization for zero field ϵ_M, which are represented as intersection points of the straight lines in fig. 1 with the ordinate.

For comparison fig. 1a shows the situation in the corresponding unfrustrated ordered case (square lattice with nearest-neighbour antiferromagnetic interaction OAFM), where ϵ_M depends linearly on M, since ϵ_M and ϵ_{M+2} differ by single spin excitations. It follows $\epsilon_M = \epsilon_0^0 + (1 - \epsilon_0^0)|m|$ and all straight lines $\epsilon(h) = \epsilon_M - mh$ intersect at a critical field $h_c = 2$. Therefore the ground state (the lowest lying line) for all $h < h_c$ belongs to the antiferromagnetic order ($\epsilon^0(\mathrm{AFM}) = -1$) and for all $h > h_c$ to the ferromagnetic one ($\epsilon^0(\mathrm{FM}) = 1 - h$).

Frustration leads to an increase of the ground state energy in comparison with OAFM, which is marked by an arrow in fig. 1b. Moreover, it also influences the M

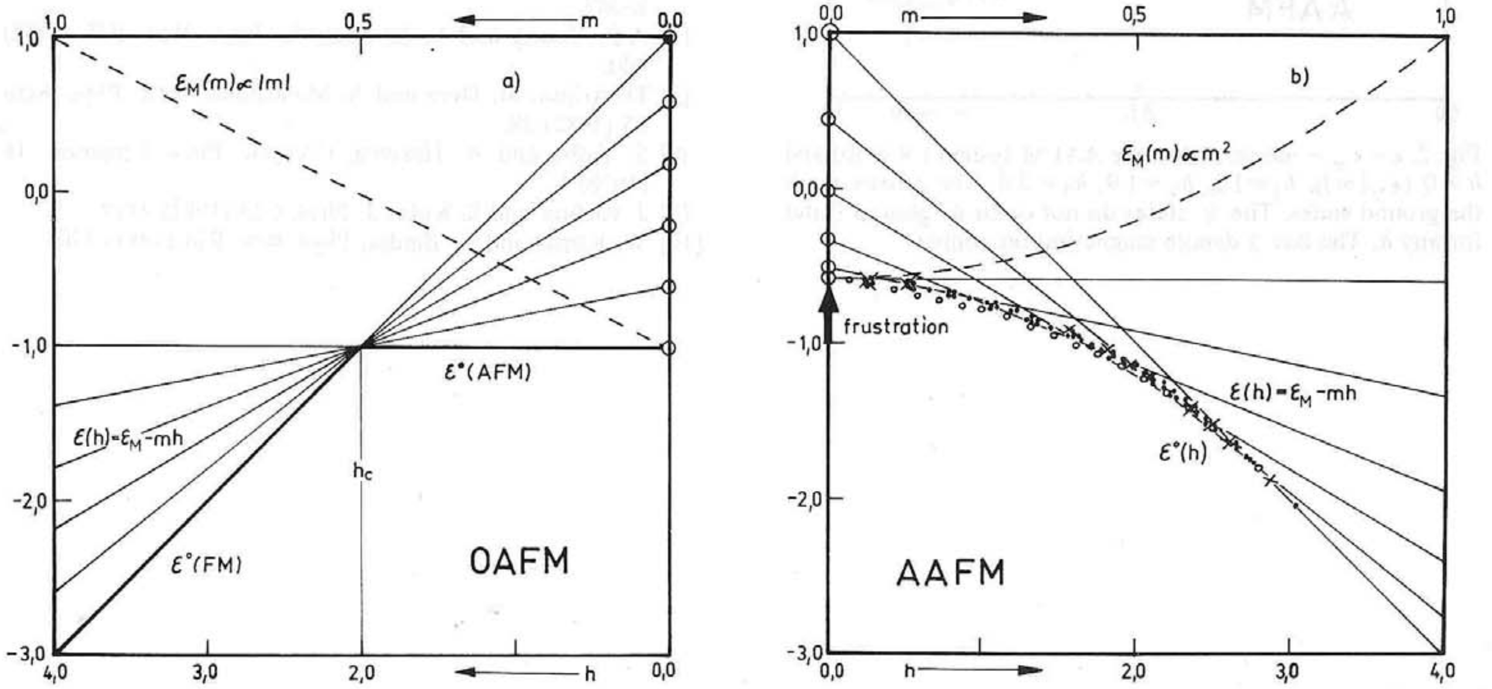

Fig. 1. Field dependence of the ground state energy $\epsilon^0(h)$ for (a) ordered square antiferromagnet OAFM (thick line); (b) computer simulated amorphous short-range antiferromagnetic systems AAFM: ○ $N = 10$, ● $N = 30$ (5 systems), × $N = 40$; the lowest states of a given magnetization in zero field ϵ_M vs. m (upper abscissa) is shown by broken lines.

dependence of the ϵ_M values the stronger the lower ϵ_M lies. For the AAFM our simulations suggest

$$\epsilon_M = \epsilon_0^0 + \left(1 - \epsilon_0^0\right) m^2 \pm \dots \quad (2)$$

Again, the lowest of the straight lines $\epsilon(h) = \epsilon_M - mh$ determine the ground state energy for a given h. In consequence of the approximation (2) $\epsilon^0(h) \propto h^2$ and $m \propto h$ results.

Fig. 2 shows $\epsilon(m)$ with h as parameter, where the minima of the curves belong to the ground state. The flat minima for intermediate fields in connection with fluctuations in ϵ_M due to *disorder* lead to metastability: States with different magnetization are nearly of the same energy. Consequently, changes of h in this region cause jumps of magnetization (fig. 3), that means the magnetization process is a sort of "devil's staircases", cf. ref. [10].

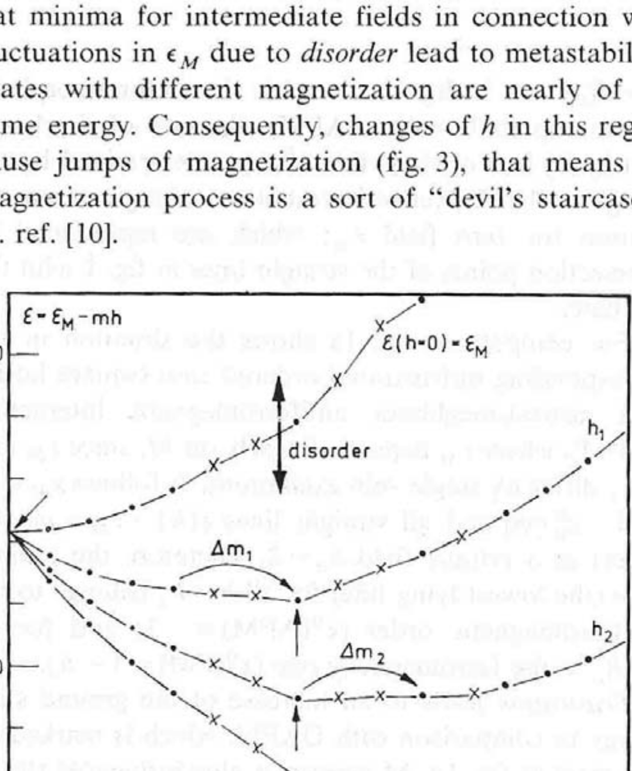

Fig. 2. $\epsilon = \epsilon_M - mh$ vs. m for one AAFM system ($N = 30$) and $h = 0$ ($\epsilon_M(m)$), $h_1 = 1.2$, $h_2 = 1.9$, $h_3 = 2.4$. The arrows mark the ground states. The × states do not occur as ground states for any h. The Δm_i's denote magnetization jumps.

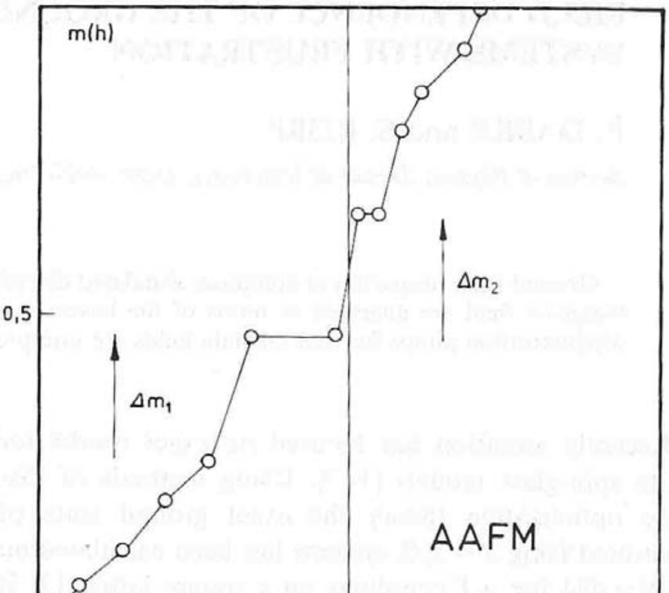

Fig. 3. Magnetization vs. field for the AAFM system ($N = 30$) of fig. 2.

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