Relationship between energy landscape and low-temperature dynamics of $\pm J$ spin glasses

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Clusters and valleys in the exact low-energy landscape of finite Edwards-Anderson $\pm J$ spin glasses are related to the distribution of spin domains and free spins in the ground states. The time evolution of the spin correlation function reflects a walk through the landscape at a given temperature and shows typical glassy behaviour.

Keywords: Spin glass; Energy landscape; Relaxation; Computer simulation

The Edwards-Anderson $\pm J$ spin glass is described by the Hamiltonian

$$H = -\sum_{i<j} J_{ij} S_i S_j \qquad (S_i = \pm 1) \tag{1}$$

on a simple cubic lattice of the size $N = L \times L \times L$ with periodic boundary conditions, where J_{ij} are coupling constants between nearest-neighbour spins S_i. The sample is prepared by randomly assigning $J_{ij} = +J$ and $-J$ to the edges of the lattice with equal probability.

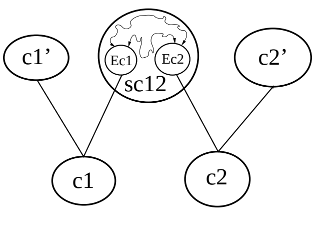

Figure 1. Schematic picture of the low-energy landscape ($L = 4$, cf. Fig. 2). Numbers of states in c1, c2, sc12, Ec1, Ec2: 12; 18; 569; 32; 120.

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For $L = 4$ all 1635796 states up to the third excitation are calculated using the branch-and-bound optimization algorithm [1,2].

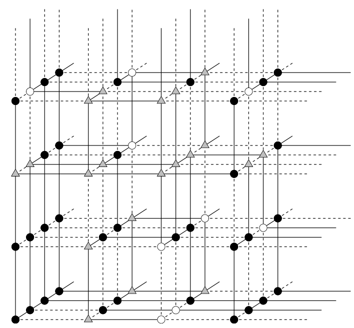

Figure 2. Two spin domains ($\bullet, \triangle$) and free spins ($\circ$) of the ground states of a sample ($L = 4$); $J_{ij} = +J$ (solid lines) and $-J$ (dotted lines).

Assuming that two neighbouring states differ in the orientation of one spin (i.e. the Hamming distance $h_d = 1$), the states form a landscape in the N-dimensional configuration space: Neighbouring states with the same energy build a cluster. Two clusters of different energies are connected, if $h_d = 1$ for at least two of their states, whereas the transition between clusters with the same energy goes over energetic barriers, which is realised in our model by saddle clusters.

Some elements of the structure of the landscape are schematically shown in Fig. 1, where mirror states are omitted. Two ground-state clusters c1 and c2 are connected by the saddle cluster sc12 in the first excitation, whereas the cluster c1'(c2') is connected *only* with c1(c2) and belongs to a valley, which is attributed to c1(c2). It should be emphasised that the internal structure of the saddle cluster sc12 is inhomogeneous. Only the states in the subsets Ec1 and Ec2 have direct connection with the corresponding ground state clusters c1 and c2, respectively. It has been shown, that the mean value $\overline{h}_d$ between Ec1 and Ec2 for our sample is about 20 and that the frequency of configurations inbetween is reduced [2]. Therefore, a dynamical transition between both valleys will be slowed down.

The relation between the energy landscape and the real-space structure can be recognised in Fig. 2. The splitting up of all ground states into clusters is caused by the existence of spin domains (i.e. subsets of all spins, which have a rigid orientation to each other [3,4]). The reversal of one spin domain in relation to another one would lead to a state with the same (ground-state) energy. Consequently, both states belong to different clusters in the configuration space. The region between the spin domains consists of free spins, which can be reversed without changing the energy. They can be assigned to the adjacent spin domains ad libitum and are responsible for the number of states of a given cluster.

The dynamical properties of the system are reflected by a random walk in the energy landscape. Especially, the process of transition between different valleys is attributed to the glassy dynamics at low temperatures. In Fig. 3 the time evolution of the spin correlation function $q(t) = < S_i^G(0) S_i(t) > /N$ is shown for a sample with $L = 12$, where $S_i^G(0)$ is the i-th spin of the ground state configuration. Instead of the normally used Metropolis Monte Carlo algorithm, the waiting time method (WTM) [5] is applied. It belongs to the class of rejectionless or "event driven" algorithms and is better suited for simulations at low temperatures.

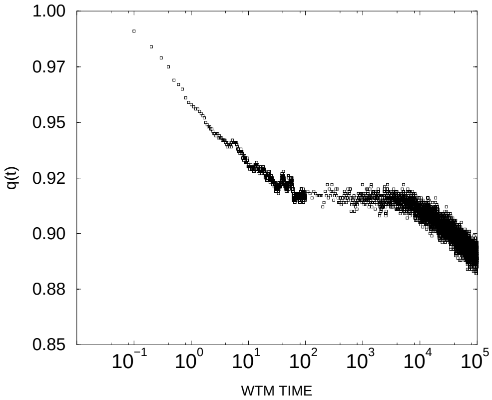

Figure 3. The spin correlation function q vs. WTM-time t for 20 runs starting from one ground state of a $L = 12$ system ($T = 0.37$).

The feature of $q(t)$ reflects the typical dynamical behaviour of glassy systems, where the plateau can be associated with a walk inside the valley (β process) followed by a further decay due to the escape process from the valley. Furthermore, using the relation $q_{pl} = 1 - 2\overline{h}_d/N$ [2], the mean Hamming distance $\overline{h}_d$ between all pairs of states in a valley can be estimated from the plateau value q_{pl} of the spin correlation function (e.g. $\overline{h}_d = 73 \pm 2$ results from $q_{pl} = 0.915 \pm 0.02$ of Fig. 3).

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REFERENCES

1. T. Klotz and S. Kobe, Acta Phys. Slovaca 44 (1994) 347.
2. J. Krawczyk and S. Kobe, Physica A 315 (2002) 302.
3. J.F. Valdes, J. Cartes, E.E. Vogel, S. Kobe, T. Klotz, Physica A 257 (1998) 557.
4. G. Hed, A.K. Hartmann, D. Stauffer, E. Domany, Phys. Rev. Lett. 86 (2001) 3148.
5. J. Dall and P. Sibani, Comp. Phys. Commun. 141 (2001) 260.