Noise spectra of ac-driven quantum dots: Floquet master-equation approach

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We study the transport properties of a quantum dot driven by either a rotating magnetic field or an ac gate voltage using the Floquet master-equation approach. Both types of ac driving lead to photon-assisted tunneling where quantized amounts of energy are exchanged with the driving field. It is found that the differential-conductance peak due to photon-assisted tunneling does not survive in the Coulomb-blockade regime when the dot is driven by a rotating magnetic field. Furthermore, we employ a generalized MacDonald formula to calculate the time-averaged noise spectra of ac-driven quantum dots. Besides the peak at zero frequency, the noise spectra show additional peaks or dips in the presence of an ac field. For the case of an applied ac gate voltage, the peak or dip position is fixed at the driving frequency whereas the position changes with increasing amplitude for the case of a rotating magnetic field. Additional features appear in the noise spectra if a dc magnetic field is applied in addition to a rotating field. In all cases, the peak or dip positions can be understood from the energy differences of two available Floquet channels.

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I. INTRODUCTION

Quantum conductors based on single molecules or semiconductor quantum dots are promising building blocks for future electronics and model systems for the study of fundamental quantum phenomena.1 However, much information on quantum conductors is beyond the reach of measurements of the current or conductance alone. Instead, the understanding of the transport properties calls for a study of the full counting statistics.2–11 In the past decade, valuable information on microscopic details of the charge transport has been obtained from measurements of the current fluctuations or current noise.12 Previous studies have shown that one can extract parameters such as the average backscattered charge, the intrinsic time scales, and the asymmetry of the dot-lead coupling from current-noise measurements. Most studies of the current noise have focused on the zero-frequency noise power $S(0)$.$^{18–20}$ The zero-frequency noise reflects the average properties of the tunneling. Since the finite-frequency current noise $S(\omega)$ is a measure of the correlations between tunneling events with their time difference conjugate to the frequency $\omega$, it is interesting to go beyond the zero-frequency limit. Aguado and Brandes have demonstrated that the noise spectra can show dip structures at the splitting energy of an open quantum two-level system, with their width controlled by its dissipative dynamics. In many works, the MacDonald formula has been used to study the current-noise spectra.$^{23–25}$

Effective in situ manipulation of quantum conductors is a key step for further development. Using a time-dependent field to manipulate the dynamics of quantum dots promises to be advantageous in situations ranging from photon-assisted inelastic tunneling to quantum pumping. When the conductor is driven by an ac field, one expects novel features due to the interplay of intrinsic oscillation frequencies and the external driving frequency. Several recent studies indicate that key information is hidden in the noise spectra of the ac-driven transport. For instance, Barrett and Stace have proposed to extract the characteristic time scales such as the inverse dephasing and relaxation rates of a solid-state charge qubit coupled to a microwave field from the noise spectrum. Wabnig et al. have proposed to estimate the coherence time of the spin in a quantum dot by measuring its noise spectra under an ac magnetic field. These results are obtained based on the rotating-wave approximation and usually in the limit of infinite on-site Coulomb interaction.

For periodically driven systems, an appropriate theoretical tool to go beyond the rotating-wave approximation is the Floquet theorem. Various attempts have been made by generalizing the existing steady-state transport approaches such as the scattering matrix and nonequilibrium Green’s functions with the help of the Floquet theorem. However, these methods are not adequate to fully take the Coulomb blockade in quantum dots into account, which dominates the transport properties of small-size quantum conductors. The quantum master equation in its various manifestations is able to give a good account of the Coulomb blockade in the weak-tunneling limit. This method has previously been generalized using the Floquet theorem to study the current and the zero-frequency noise power in an ac-driven conductor.

In the present study, we employ the Floquet master equation in the Fock space of an ac-driven quantum dot to study the transport properties such as the differential conductance and the full noise spectrum in the sequential-tunneling limit. As the ac field we consider a rotating magnetic field as well as an ac gate voltage for comparison. We employ a generalized MacDonald formula for the time-averaged noise spectra in the presence of a periodic ac field. An equivalent form of the generalized MacDonald formula has been given by Clerk and Girvin without derivation. For completeness, we present a derivation in the Appendix. Note that the authors
are concerned with a different case, namely, an ac bias voltage and do not employ the Floquet formalism.

Our paper is organized as follows. In Sec. II, a Floquet master-equation formalism is presented to study the transport properties of ac-driven quantum dots. Expressions for the noise spectra are derived based on the full counting statistics and the generalized MacDonald formula. In Sec. III, the transport properties of the ac-driven quantum dot are studied. The ac field is either a rotating magnetic field or an oscillating on-site energy due to a periodic gate voltage. The characteristic features in the transport properties are presented and discussed. In Sec. IV, a brief summary is given.

II. FORMALISM

A. Model

In this paper, we study the transport properties of a single-level quantum dot driven by an ac field. The quantum dot is coupled to the left (L) and right (R) electron leads. The leads are assumed to be ideal and free of interactions. The Hamiltonian of the model system can be written as

\[ H(t) = H_L + H_R + H_{dot}(t) + H_T = H_0 + H_T, \]

where \( H_{dot} \) is the Hamiltonian of the isolated quantum dot, which contains the effects of the ac field and the Coulomb interaction. \( H_L = \sum_{\alpha} \varepsilon_{\alpha} c_{\alpha L} \) represents the Hamiltonian of lead \( L \), where \( c_{\alpha L} \) annihilates (creates) an electron with spin \( \sigma \), crystal momentum \( \mathbf{k} \), and energy \( \varepsilon_{\alpha} \) in lead \( L \), and \( H_R = \sum_{\alpha} \varepsilon_{\alpha} c_{\alpha R} \) describes the coupling between the quantum dot and the leads, where \( d_{\alpha} \) (\( d_{\alpha}^\dagger \)) is the spin-\( \sigma \) electron creation (annihilation) operator in the quantum dot. We note that we do not assume an infinite Coulomb-interaction strength \( U \rightarrow \infty \), in contrast to previous studies. Instead, finite Coulomb interaction will be included by taking the doubly occupied state into account.

In the following, we focus on the limit of weak dot-lead coupling and investigate the transport properties of the quantum dot using the Floquet master-equation method. For a small quantum dot, for which the Coulomb interaction can dominate the transport behavior, the treatment of the Coulomb interaction must go beyond the mean-field level. To this end, it is convenient to rewrite the dot Hamiltonian in the electron-number basis of the Fock space. In this description, the quantum dot can either be in the empty state \( |0\rangle \), the singly occupied state \( |\uparrow\rangle \) or \( |\downarrow\rangle \), or the doubly occupied state \( |\uparrow\downarrow\rangle \). In the following, we denote the states in the Fock space by Latin letters, \( |a\rangle \) for the empty state \( |0\rangle \) or \( |\downarrow\rangle \) and the fully occupied state \( |\uparrow\uparrow\rangle \). Using this orthonormal basis, the dot-lead coupling can be described naturally with the help of Hubbard operators,

\[ X_{ab} = |a\rangle \langle b|, \]

which describes the transition of the quantum dot from state \( |b\rangle \) to state \( |a\rangle \). The second-quantized dot-electron creation operator can thus be rewritten in terms of the Hubbard operators as

\[ d_{\sigma} = |\sigma\rangle \langle 0| + \eta_\sigma \langle \uparrow| |\sigma\rangle, \]

where \( \eta_\sigma \) represents the opposite spin of \( \sigma \) and the factor \( \eta_\sigma = \pm 1 \) for \( \sigma = \uparrow, \downarrow \), respectively, is due to the anticommutation relation of the fermions. In terms of these Hubbard operators, the Hamiltonian for the dot-lead coupling and the isolated dot can be rewritten as

\[ H_{dot} = \sum_{ab} H_{ab}^0 |a\rangle \langle b|, \]

\[ H_T = \sum_{ab,\mathbf{k}\sigma} V_{ab}^{\mathbf{k}\sigma} X_{ab} + \text{H.c.}, \]

respectively. Here, we have made the coupling strength \( V_{ab}^{\mathbf{k}\sigma} \) depend on the occupancy of the initial and final states of the transition in the quantum dot. The explicit form of the dot Hamiltonian depends on the details of the device geometry and the external ac-driving field. It will be specified in the following sections.

B. Floquet quantum-master-equation approach

1. Floquet states

Due to the presence of a time-periodic external field, the dynamics of the quantum dot is governed by a Hamiltonian that is periodic in time with the frequency \( \Omega = 2\pi/T \), i.e., \( H_{dot}(t) = H_{dot}(t + T) \), where \( T \) denotes the period. The solution of the time-periodic Hamiltonian can be simplified by the Floquet theorem, which states that the solution of the Schrödinger equation for the dot Hamiltonian can be obtained from \( \psi(t) = \psi_0 \psi(t) \), with the time-independent Floquet quasienergy and \( |\psi(t)\rangle \) is the corresponding Floquet state, which has the same period \( T \) \( |\psi(t)\rangle = |\psi(t + T)\rangle \). Here, Greek letters are used to denote the Floquet states. Further simplification is possible by decomposing the Floquet states into a Fourier series,

\[ |\psi(t)\rangle = \sum_k e^{-ik\Omega t} |\psi_k\rangle \]

with the reverse transformation

\[ |\psi_k\rangle = \frac{1}{T} \int_0^T e^{ik\Omega t} |\psi(t)\rangle dt \]

and analogously for \( H_{dot}(t) \). The Fourier transform of Eq. (6) then reads

\[ \sum_k H_{\text{dot},k} e^{-ik\Omega t} |\psi_k\rangle = k\Omega |\psi_k\rangle + e_\sigma |\psi_k\rangle. \]

The quasienergy \( e_\sigma \) can evidently be restricted to the first Brillouin zone \( [0, \Omega] \) of the Floquet space while the Floquet index \( k \) can assume any integer value. Equivalently, we can view \( e_\sigma + k\Omega \) as the quasienergy in the extended zone scheme.

We also introduce the Hubbard operator in the Floquet states to describe the transition between the Floquet states as
\(X_{\alpha\beta}(t) = |\alpha(t)\rangle\langle\beta(t)|\). For the time-dependent transport, it is more convenient to work with these Floquet states. This is most advantageous in transformations of the following form, which we will use in the derivation below,

\[
\tilde{X}_{\alpha\beta}(t', t) = U_{0}(t', t)X_{\alpha\beta}(t')U_{0}^{\dagger}(t', t) = e^{i\epsilon_{\alpha\beta}(t) - i\epsilon_{\alpha\beta}(t')}X_{\alpha\beta}(t),
\]

where

\[
U_{0}(t', t) = T_{c} \exp \left( -i \int_{t}^{t'} dt' [H_{L} + H_{R} + H_{dot}(t')] \right)
\]

denotes the time-evolution operator due to the Hamiltonian in the absence of tunneling. Here, \(T_{c}\) is the time-ordering operator and the dot Hamiltonian is explicitly time dependent.

2. Floquet quantum master equation with counting fields

For a quantum dot coupled to external leads, the exact master equation can be written in the interaction picture as

\[
d\frac{d}{dt} \rho(t) = -i[H_{F}(t), \rho_{\text{tot}}(t)] - \int_{0}^{t} dt'[H_{T}(t'), [H_{T}(t'), \rho_{\text{tot}}(t')]],
\]

where \(H_{F}(t) = U_{0}^{\dagger}(t, t_{0})A(t)U_{0}(t, t_{0})\) denotes an operator in the interaction picture and \(\rho(t)\) is the density matrix in the Fock space of the full system.

A complete description of the electronic transport through the quantum dot is provided by the full counting statistics. Properties such as the noise spectrum are determined by the counting statistics of the electrons arriving at and departing from the leads. All information on the counting statistics is contained in the moment-generating function \(\phi(\chi_{L}, \chi_{R}) = \langle \exp(\chi_{L} N_{L} + \chi_{R} N_{R}) \rangle\). Here, \(\chi_{L}\) represents the counting field in the lead \(L\), which counts how many electrons have tunneled into or out of the lead. \(N_{i} = \sum_{n} c_{i,n}^{\dagger} c_{i,n}\) is the electron-number operator in lead \(L\). We introduce the operator

\[
\mathcal{F}(\chi_{L}, \chi_{R}; t) = \text{Tr}_{\text{leads}} e^{i\chi_{L} N_{L} + i\chi_{R} N_{R}} \rho(t).
\]

In this we follow Kaiser and Kohler, except that we introduce two counting fields. In the limit of \(\chi_{L} \to 0\) and \(\chi_{R} \to 0\), \(\mathcal{F}\) becomes the reduced density matrix of the quantum dot, \(\rho_{\text{dot}} = \text{Tr}_{\text{leads}} \rho\). Moreover, the moment-generating function \(\phi(\chi_{L}, \chi_{R}; t)\) can be obtained by tracing out the dot degrees of freedom, \(\phi = \text{Tr}_{\text{dot}} \mathcal{F}\). We decompose \(\mathcal{F}\) into a Taylor series,

\[
\mathcal{F} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(i\chi_{L})^{m}(i\chi_{R})^{n}}{m!n!} \mathcal{F}^{m,n},
\]

where the coefficients

\[
\mathcal{F}^{m,n} = \frac{\mathcal{F}^{m+n}}{\mathcal{F}^{(i\chi_{L})^{m}(i\chi_{R})^{n}}} |_{\chi_{L}, \chi_{R} = 0} = \text{Tr}_{\text{leads}} N_{L}^{m} N_{R}^{n}
\]

provide a direct access to the moments \(\langle N_{L}^{m} N_{R}^{n} \rangle = \text{Tr}_{\text{dot}} \mathcal{F}^{m,n}\). In particular, we obtain the reduced density matrix of the quantum dot, \(\rho_{\text{dot}} = \mathcal{F}^{0,0}\).

To find the solutions for \(\mathcal{F}\), we first transform the equation of motion for the density matrix in the interaction picture, Eq. (12), back to the Schrödinger picture,

\[
d\frac{d\rho(t)}{dt} + i[H_{0}(t), \rho(t)] = -i[H_{T}, U_{0}^{\dagger}(t_{0}, t)\rho(t_{0})U_{0}(t_{0}, t)]
\]

\[
- \int_{t_{0}}^{t} dt'[H_{T}, U_{0}^{\dagger}(t', t)] \times[H_{T}, \rho(t')]U_{0}(t', t).
\]

Then, we multiply by \(e^{i\chi_{L} N_{L} + i\chi_{R} N_{R}}\) from the left and take the trace over the lead degrees of freedom to obtain

\[
d\frac{d\mathcal{F}(\chi_{L}, \chi_{R}; t)}{dt} + i[H_{\text{dot}}(t), \mathcal{F}(\chi_{L}, \chi_{R}; t)]
\]

\[
= -i \text{Tr}_{\text{leads}} e^{i\chi_{L} N_{L} + i\chi_{R} N_{R}}[H_{T}, U_{0}^{\dagger}(t_{0}, t)\rho(t_{0})U_{0}(t_{0}, t)]
\]

\[
- \int_{t_{0}}^{t} dt' \text{Tr}_{\text{leads}} e^{i\chi_{L} N_{L} + i\chi_{R} N_{R}}[H_{T}, U_{0}^{\dagger}(t', t)] \times[H_{T}, \rho(t')]U_{0}(t', t),
\]

which is still exact.

We now assume that the full density operator is of product form at the initial time \(t_{0}\), \(\rho(t_{0}) = \rho_{\text{dot}}(t_{0}) \otimes \rho_{\text{leads}}\), where \(\rho_{\text{leads}}\) describes the leads in separate thermal equilibrium. This assumption is reasonable since we are not interested in transient effects coming from the initial state. Such effects have been studied by Flindt et al. The first term on the right-hand side of Eq. (17) then vanishes. Furthermore, we make the sequential-tunneling approximation appropriate for weak tunneling, i.e., we treat the tunneling perturbatively to second order in \(H_{T}\). Since two powers of \(H_{T}\) are already explicit in the second term on the right-hand side of Eq. (17), we can express \(\rho(t')\) in terms of the unperturbed time evolution, \(\rho(t') = U_{0}^{\dagger}(t', t_{0})\rho(t_{0})U_{0}(t_{0}, t')\). This makes the master equation local in time, i.e., Markovian. For details, see, e.g., Ref. 39. We thus do not include non-Markovian effects as studied by Flindt et al. This is valid if the relaxation time in the leads is short compared to the typical time scales of the dot, which in our case include the period of the ac field. Since relaxation times in metallic leads are on the order of femtoseconds, this is easily satisfied.

Finally, we send \(t_{0} \to -\infty\) and obtain

\[
d\frac{d\mathcal{F}(\chi_{L}, \chi_{R}; t)}{dt} + i[H_{\text{dot}}(t), \mathcal{F}(\chi_{L}, \chi_{R}; t)]
\]

\[
= -\int_{0}^{\infty} d\tau \text{Tr}_{\text{leads}} e^{i\chi_{L} N_{L} + i\chi_{R} N_{R}}[\tilde{H}_{T}(\tau, t), \rho(t)],
\]

where \(\tilde{H}_{T}(\tau, t) = U_{0}^{\dagger}(t', t)\tilde{H}_{T}U_{0}(t', t)\).

To obtain the Floquet master equation, we write Eq. (18) in the basis of Floquet states \(|\alpha(t)\rangle\) and \(|\beta(t)\rangle\). By making use of the relation Eq. (10) and tracing out the lead degrees of freedom, we arrive at the equation of motion for \(\mathcal{F}\),
where the superoperators are given by

\[
\frac{d}{dt} \mathcal{F}_{\alpha \beta}(\chi_L, \chi_R, t) = \left\{ [\mathcal{L} + (e^{i\chi_L} - 1) \mathcal{J}_L + (e^{-i\chi_L} - 1) \mathcal{J}_L - (e^{i\chi_R} - 1) \mathcal{J}_R + (e^{-i\chi_R} - 1) \mathcal{J}_R] \mathcal{F}(\chi_L, \chi_R, t) \right\}_{\alpha \beta},
\]

(19)

and

\[
\begin{align*}
\langle \mathcal{J}_L \mathcal{F} \rangle_{\alpha \beta} &= -i(e_\alpha - e_\beta) \mathcal{F}_{\alpha \beta} \frac{1}{2\pi} \int_0^\infty d\tau \int ds \sum_{ab, mn} \sum_{\gamma \delta} \sum_{\imath=L,R} \left\{ \left[ f_\delta(e) e^{i\tau \mathcal{T}\mathcal{T}_{ab,mn}} \right] \mathcal{F}_{\gamma \delta}(\chi_L, \chi_R, t) \right\} \\
&\quad \times e^{i(e_\alpha - e_\beta)\tau} \left( \langle \mathcal{F}(\chi_L, \chi_R, t) \mathcal{F}(\chi_L, \chi_R, t) \rangle \langle \mathcal{F}(\chi_L, \chi_R, t) \mathcal{F}(\chi_L, \chi_R, t) \rangle \right),
\end{align*}
\]

(20)

\[
\begin{align*}
\langle \mathcal{J}_R \mathcal{F} \rangle_{\alpha \beta} &= -i(e_\alpha - e_\beta) \mathcal{F}_{\alpha \beta} \frac{1}{2\pi} \int_0^\infty d\tau \int ds \sum_{ab, mn} \sum_{\gamma \delta} \sum_{\imath=L,R} \left\{ \left[ f_\delta(e) e^{i\tau \mathcal{T}\mathcal{T}_{ab,mn}} \right] \mathcal{F}_{\gamma \delta}(\chi_L, \chi_R, t) \right\} \\
&\quad \times e^{i(e_\alpha - e_\beta)\tau} \left( \langle \mathcal{F}(\chi_L, \chi_R, t) \mathcal{F}(\chi_L, \chi_R, t) \rangle \langle \mathcal{F}(\chi_L, \chi_R, t) \mathcal{F}(\chi_L, \chi_R, t) \rangle \right),
\end{align*}
\]

(21)

Here, we have defined the tunneling rate \( T_{mn,ab}(\epsilon) = 2\pi \rho(\epsilon) V_{mn,ab}^2 V_{1k,\alpha}^2 \), where \( \rho(\epsilon) \) is the density of states in lead \( l \).

Inserting the Taylor expansion of \( \mathcal{F} \) in Eq. (14) into its equation of motion [Eq. (19)], one obtains a hierarchy of equations for the expansion coefficients,

\[
\frac{d}{dt} \mathcal{F}^{0.0} = \mathcal{L} \mathcal{F}^{0.0},
\]

(23)

\[
\frac{d}{dt} \mathcal{F}^{1.0} = \mathcal{L} \mathcal{F}^{1.0} + (\mathcal{J}_L - \mathcal{J}_L) \mathcal{F}^{0.0},
\]

(24)

\[
\frac{d}{dt} \mathcal{F}^{0.1} = \mathcal{L} \mathcal{F}^{0.1} + (\mathcal{J}_R - \mathcal{J}_R) \mathcal{F}^{0.0},
\]

(25)

\[
\frac{d}{dt} \mathcal{F}^{2.0} = \mathcal{L} \mathcal{F}^{2.0} + 2(\mathcal{J}_L - \mathcal{J}_L) \mathcal{F}^{1.0} + (\mathcal{J}_L + \mathcal{J}_L) \mathcal{F}^{0.0},
\]

(26)

\[
\frac{d}{dt} \mathcal{F}^{0.2} = \mathcal{L} \mathcal{F}^{0.2} + 2(\mathcal{J}_R - \mathcal{J}_R) \mathcal{F}^{1.0} + (\mathcal{J}_R + \mathcal{J}_R) \mathcal{F}^{0.0},
\]

(27)

etc. As described above, these coefficients contain the full counting statistics. The charge current out of lead \( l \) is defined as the negative of the time derivative of the charge in lead \( l \), \( I_l(t) = e dN_l/dt \). The final expression for the current out of the left lead is given by

\[
\langle I_L(t) \rangle = e \text{ Tr}_{\text{dot}}(\hat{J}^{0.1}) = e \text{ Tr}_{\text{dot}}(\mathcal{J}_L - \mathcal{J}_L) \mathcal{F}^{0.0},
\]

(28)

The dc component of the current then gives the time average \( \bar{I} \). In order to find \( \mathcal{F}^{0.0} \), we have to solve a set of linear equations with the help of the normalization condition of probability \( \text{Tr}_{\text{dot}} \mathcal{F}^{0.0}(t) = 1 \).

3. Generalized MacDonald formula for time-averaged noise spectra

We are interested in the frequency-dependent current noise of the quantum dot driven by an ac field. The zero-frequency current noise for nonadiabatical driving has been investigated in Ref. 44 using the Floquet master-equation approach in the Coulomb-blockade regime. The symmetrized current-current correlation function is defined by

\[
S_{II}(t, t') = \langle \hat{I}_L(t) \hat{I}_L(t') \rangle - 2 \langle \hat{I}_L(t) \rangle \langle \hat{I}_L(t') \rangle,
\]

(30)

where \( \hat{I}_L(t) \) represents the current operator at the time \( t \) from the lead \( l \). The current-noise spectra are defined as the Fou-
rier transform of $S_{II}(t,t')$. Since our system is driven by an ac field, the current noise is a double-time function. However, the periodicity of our problem makes it possible to characterize the spectra by averaging over one driving period.

At finite frequencies, the total current $I(t)$ measured by a measurement device depends on both the particle and the displacement currents in the lead-dot-lead junction. If one expresses the displacement currents by the particle currents $I_L$ and $I_R$, one obtains the Ramo-Shockley theorem,\textsuperscript{12,50,51}

$$I(t) = aI_L(t) - bI_R(t).$$

Here the coefficients $a$ and $b$, which satisfy $a + b = 1$, are specified by the device geometry. It is straightforward to obtain

$$\overline{S}_l(\omega) = a^2\overline{S}_{LL}(\omega) + b^2\overline{S}_{RR}(\omega) - ab[\overline{S}_{LR}(\omega) + \overline{S}_{RL}(\omega)],$$

where $\overline{S}_{ll}(\omega)$ $(l,l' = L,R)$ represents the frequency-dependent time-averaged current correlation between $I_l$ and $I_{l'}$,

$$\overline{S}_{ll}(\omega) = \frac{1}{T} \int_0^T dt'd e^{i\omega(t-t')} \overline{S}_{ll}(t,t').$$

In this study, we used two counting fields to derive the noise spectra. An alternative approach is to calculate the charge fluctuation on the dot employing the quantum regression formula.\textsuperscript{3,14,52,53} The two approaches are physically equivalent due to the charge conservation condition in the transport.

The formula for the zero-frequency noise has been presented in Ref. \textsuperscript{44}. For the two-terminal device, it is adequate to find the time-averaged zero-frequency noise from the fluctuations of the current flowing out of a chosen lead, $\overline{S}(0) = \overline{S}_{ll}(0)$. The solution for $\overline{S}(0)$ resulting from the Floquet quantum master equation reads

$$\overline{S}(0) = \frac{2}{T} \int_0^T dt' e^{i\omega(t-t')} \overline{S}_{ll}(t,t'),$$

where the prefactor of 2 is inserted to make the noise formula consistent with Ref. \textsuperscript{12}. For Poissonian noise, we then obtain $\overline{S}(0) = 2e^2T$. $\delta_1$ is the usual Kronecker symbol. Following Ref. \textsuperscript{44}, the new function $\mathcal{F}_{0,0}$ in the noise expression is defined as

$$\mathcal{F}_{0,0} = \mathcal{F}_{0,0} - \mathcal{F}_{0,0} \mathcal{L} \mathcal{L} \mathcal{F}_{0,0},$$

and satisfies the equation of motion

$$\mathcal{F}_{0,0} = \mathcal{L} \mathcal{F}_{0,0} + \left( \mathcal{J}_{1+} - \mathcal{J}_{1-} \right) \mathcal{F}_{0,0}.$$
it is customary to talk about photon-assisted processes in this context, the treatment of the electromagnetic field in the Hamiltonian is completely classical.

We work in the sequential-tunneling regime and choose a symmetric-coupling geometry with $a=b$. We assume that the bias voltage $V_{ac}$ symmetrically shifts the chemical potentials by $\mu_{L,R}=\pm eV_{dc}/2$. In the framework of wideband approximation, the tunneling rate is given by $\Gamma_{mnij}(e)=2\pi tV_{ij}^\dagger V_{mn}^\text{tunnel}$ where we have assumed the coupling strength $V_{ij}^\dagger V_{mn}^\text{tunnel}$ to be a constant and have set the density of states of lead $l$ to unity. In the present study, we have assumed the tunneling matrix to be independent of the energy and the occupation number on the dot. An inclusion of state-dependent tunneling is straightforward. We assume that the electrons tunneling in and out of the dot with an energy-independent rates $\Gamma=\Gamma_{mnij}(e)$ and set $\Gamma=1$ as the energy unit.

### A. Differential conductance

We start our discussion with the differential conductance. The gray-scale plot Fig. 1 shows the differential conductance $dI/dV_{dc}$ vs the dc bias voltage $V_{dc}$ and the gate voltage $V_G$ with or without an ac field. The calculations are for the Coulomb interaction strength $U=24$ and the temperature $k_B T =0.32$. The frequency of the ac field is $\Omega=8$.

Without an ac field, Fig. 1(a) gives the familiar diamond structure due to the Coulomb blockade. Numerical results for the differential conductance when the quantum dot is modulated by an ac gate voltage are presented in Fig. 1(b). Figure 1(c) gives the results when the quantum dot is modulated by a rotating magnetic field. Figure 1(d) shows the differential conductance when the quantum dot is modulated by a rotating magnetic field in the $xy$ plane while a dc magnetic field is applied in the $z$ direction, i.e., in the plane of the rotating magnetic field.

When there is an ac field, several striking features emerge in the differential conductance: (1) at the edge of the Coulomb diamond, the sharp differential-conductance peak for the dc transport shown in Fig. 1(a) is partially suppressed by the ac gate voltage or the rotating magnetic field. Note the different gray scales in Figs. 1(a)–1(d). This can be attributed to the suppression of the elastic resonant peak by the photon-assisted processes. (2) In the presence of an ac field, there are lines parallel to the edges of the Coulomb diamond. The distance of these lines to the peak position is approximately the frequency of the ac field, indicating a photon-assisted tunneling process. (3) An interesting feature of these lines can be observed inside the Coulomb diamonds. For the ac gate voltage, the Floquet quasienergies are spin degenerate. Therefore, the main lines in the differential-conductance plot in Fig. 1(b) are not split. However, satellites due to photon-assisted inelastic tunneling events appear in which an energy quantum of $\Omega$ is absorbed from or emitted into the driving field. We see from Fig. 1(b) that these additional lines remain distinct inside the Coulomb diamond. On the other hand, when the quantum dot is driven by a rotating magnetic field, the quasienergies are not degenerate. Therefore, the main elastic lines are split into two at the edge of the Coulomb

\[
\mathcal{F}^{0,1} = (S - L)^{-1} (J_{1+} - J_{1-}) \mathcal{F}^{0,0},
\]

\[
\mathcal{F}^{2,0} = (S - L)^{-1} [2(J_{1+} - J_{1-}) \mathcal{F}^{1,0} + (J_{1+} + J_{1-}) \mathcal{F}^{0,0}],
\]

\[
\mathcal{F}^{0,2} = (S - L)^{-1} [2(J_{1+} - J_{1-}) \mathcal{F}^{0,1} + (J_{1+} + J_{1-}) \mathcal{F}^{0,0}],
\]

\[
\mathcal{F}^{1,1} = (S - L)^{-1} [(J_{1+} - J_{1-}) \mathcal{F}^{0,1} + (J_{1+} - J_{1-}) \mathcal{F}^{1,0}],
\]

where we have omitted the arguments $s$. The solutions for these coefficients together with the generalized MacDonald formula [Eq. (37)] give the desired time-averaged current-noise spectra of the ac-driven quantum dot. The approach presented in this study can easily be generalized to take more complex structures with multiple levels and interval transitions into account.
Interestingly, the lines due to photon-assisted tunneling now only appear outside of the Coulomb diamond, as can be seen in Fig. 1c, indicating that the photon-assisted tunneling is forbidden inside the Coulomb diamond. When the quantum dot is modulated by a rotating magnetic field and a dc magnetic field is applied in the plane of the rotating field, these lines in the Coulomb-blockade regime revive. This can be clearly seen in Fig. 1d.

The disappearance of the photon-assisted tunneling inside the Coulomb diamond for a pure rotating magnetic field can be understood as follows. In the Coulomb diamond, the Floquet quasienergies corresponding to the singly occupied states are far below the Fermi energies of the two leads. A direct tunneling between the dot and the leads is forbidden due to the Pauli principle and Coulomb blockade. Therefore, an electron is effectively trapped in one quantum state on the dot. According to our previous discussion, only the spin direction of this quantum state can evolve with the rotating magnetic field. However, its eigenvalues of \( H_{\text{dot}}(t) \) remain unchanged. Therefore, the electron cannot gain extra energy from the ac field by absorbing or emitting a photon. In the Coulomb diamond, electrons on the dot are able to tunnel out via the photon-assisted tunneling and we again find the lines due to the photon-assisted differential-conductance peaks inside the Coulomb diamond as shown in Fig. 1d.

When an electron is injected from the lead into the dot, the system can absorb or emit photons, i.e., the Floquet index \( k \) can change. One could say that transport happens via several Floquet channels. Such photon-mediated tunneling can then give rise to the photon-assisted differential-conductance peaks.

The situation becomes different when a dc magnetic field is applied in the plane of the rotating magnetic field as shown in Fig. 1d. In that case, the eigenstates and the eigenvalues of \( H_{\text{dot}}(t) \) change periodically. Electrons can gain extra energy from the ac field by absorbing or emitting a photon. In the Coulomb diamond, electrons on the dot are able to tunnel out via the photon-assisted tunneling and we again find the lines due to the photon-assisted differential-conductance peaks inside the Coulomb diamond as shown in Fig. 1d.

**B. Zero-frequency Fano factor**

In the following, we show numerical results for the time-averaged zero-frequency noise of the quantum dot. The zero-frequency noise has been studied by the quantum-master-equation method in the stationary\(^{12,16,23}\) and also in the time-dependent case.\(^{44}\) Without ac field and at zero temperature, the zero-frequency Fano factor \( S(0)/2eI \) describes the devia-
FIG. 2. (Color online) Fano factor of the quantum dot as a function of dc bias voltage $V_{dc}$ in unit of $V_G$ for different amplitudes $V_{ac}$ of the ac gate voltage. The parameters of the device are given in the text.

FIG. 3. (Color online) Fano factor of the quantum dot as a function of dc bias voltage $V_{dc}$ in unit of $V_G$ for different amplitudes $B_{ac}$ of the rotating magnetic field. The other parameters are the same with those of Fig. 2.

FIG. 4. (Color online) Noise spectra for different gate-voltage amplitudes $V_{ac}$. Independent of $V_{ac}$, the main peak position is fixed at $\Omega$. The inset shows an enlarged view of the noise spectra around $2\Omega$. 

C. Frequency-dependent Fano factor

Now we present our results for the full current-noise spectra of a quantum dot under an ac field. The noise spectra have previously been studied in the stationary-transport regime. An analytical expression for the noise spectrum of a single-level quantum dot can be found in Ref. 23. Unless stated otherwise, the following calculations assume $U=24$, $V_{dc}=12.7$, $T=1.6$, $\Omega=8$, and $eV_G=-8$. We introduce the frequency-dependent Fano factor $\bar{S}(\omega)/2e\bar{I}$ to characterize the time-averaged noise power. As discussed previously, the ac gate voltage and the rotating magnetic field will modulate the quantum conductor in different ways. In the following, we show that the noise spectra are also strikingly different.

In Fig. 4, we present the results for the frequency-dependent Fano factor $\bar{S}(\omega)/2e\bar{I}$ as a function of the frequency $\omega$ for different amplitudes $V_{ac}$. No dc magnetic field is applied. Without an ac field ($V_{ac}=0$), the noise spectrum shows a peak at zero frequency and approaches a constant value for large $\omega$. The peak in the noise spectrum is due to an ac gate voltage, the Fano factor deviates from the dc behavior with increasing ac field. Additional plateaus can be observed in the Fano-factor curve when we vary the dc bias voltage. Transitions between plateaus result from additional Floquet channels becoming available.
NOISE SPECTRA OF ac-DRIVEN QUANTUM DOTS:

the elastic processes in the transport. When an ac gate voltage is applied, additional structures in the noise spectra are expected due to photon-assisted processes. For the present set of parameters, one can clearly see that with increasing amplitude of the ac gate voltage, an additional peak appears in the noise spectrum. While the height and width of this peak vary a lot with increasing amplitude, its peak position \( \omega_p \) remains almost unchanged at the external driving frequency \( \Omega \).

Now we turn to the rotating magnetic field in the \( xy \) plane. In Fig. 5, we plot the Fano factor as a function of the frequency \( \omega \) for different amplitudes \( B_{ac} \) of the rotating magnetic field. Similarly to the results presented in Fig. 4, a peak is generated and the width and height of this peak depend on the amplitude. However, the peak position is not fixed at \( \Omega \) in contrast to what we have observed in Fig. 4 for the ac gate voltage. Instead, its position shifts with increasing amplitude, as shown in Fig. 5.

By comparing Figs. 4 and 5, we see that the peak position of the noise spectra behaves differently when we increase the ac strength, depending on the type of the ac field. Recalling that when electrons tunnel through a time-independent quantum two-level system, its current-noise spectra show additional structure at the energy difference of the two transport channels of the system due to its internal coherent dynamics, we will show that the peak position of the noise spectra for ac transport can be understood from the interference between two possible Floquet transport channels. If the quantum dot is modulated by a rotating magnetic field, the last term in the dot Hamiltonian (Eq. (47)) shows that the ac magnetic field couples one spin state with the quasienergy \( \epsilon \) with a state with the opposite spin and the quasienergy \( \epsilon - \Omega \) (in the extended zone scheme). The coupling strength is given by \( B_{ac} \). The corresponding Floquet Hamiltonian then decomposes into \( 2 \times 2 \) blocks of the form

\[
h_{Fl} = \begin{pmatrix} \epsilon & B_{ac} \\ B_{ac} & \epsilon - \Omega \end{pmatrix}.
\]

The resulting quasienergies in the first Brillouin zone \([0, \Omega]\) are

\[
\epsilon_1 = \epsilon - \frac{\Omega}{2} + \frac{\sqrt{\Omega^2 + 4 B_{ac}^2}}{2},
\]

\[
\epsilon_2 = \epsilon + \frac{\Omega}{2} - \frac{\sqrt{\Omega^2 + 4 B_{ac}^2}}{2},
\]

with the difference

\[
\omega_p = \epsilon_2 - \epsilon_1 = 2 \Omega - \sqrt{\Omega^2 + 4 B_{ac}^2}.
\]

(These expressions hold if \( B_{ac} < \sqrt{\Omega/2} \).

If now an electron tunnels into the dot, the system ends up in a superposition of the two Floquet states, the phases of which change with different angular frequencies, corresponding to spin precession with the difference frequency \( \omega_p \). When the electron tunnels out again, the superposition is projected onto the spin direction of the original electron since lead electron creation and annihilation operators are paired with identical quantum numbers in the master equation. This leads to interference with a typical frequency \( \omega_p \), which enhances the current-current correlation function \( S_j(t, t') \) in Eq. (30) for \( t-t' \) being a multiple of the period \( 2 \pi/\omega_p \) and thus leads to a peak in the noise spectrum at \( \omega_p \).

The peaks seen in Fig. 5 are indeed centered at \( \omega_p \), given by Eq. (51).

Comparing with the stationary transport through a stationary two-level system, the transport through a quantum dot with rotating magnetic field can be understood as another type of two-level quantum system. The significant difference here is that our two levels are defined by the Floquet channels due to a periodic ac field and not by the true eigenenergies of an time-independent Hamiltonian.

When an ac gate voltage is applied to the quantum dot, the ac field will not couple the different spin states. Only the eigenvalues of \( H_{dot}(t) \) will be modulated, see Eq. (47). The corresponding Floquet Hamiltonian decomposes into two infinite blocks for the two spin directions, where each block has the tridiagonal form

\[
h_{Fl} = \begin{pmatrix} \epsilon + \Omega & V_{ac}/2 & 0 \\ V_{ac}/2 & \epsilon & V_{ac}/2 \\ 0 & V_{ac}/2 & \epsilon - \Omega \end{pmatrix}.
\]

Therefore, the electrons can tunnel through the quantum dot via infinitely many Floquet channels with the same quasienergy in the first Brillouin zone \([0, \Omega]\) but all possible Floquet indices \( k \). The quasienergies in the extended zone scheme thus differ by integer multiples of \( \Omega \). These quasienergy differences define the peak positions in the noise spectra. In Fig. 4, a peak at \( \Omega \) appears, corresponding to two channels with their Floquet indices (photon numbers) differing by unity. One should also expect peak structures at \( n \Omega, n > 1 \). However, to observe these peak structures, one may need a stronger ac field to enable multiphoton-assisted transport. In the inset of Fig. 4, a small shoulder emerges at \( 2 \Omega \) for the largest amplitude, \( V_{ac} = 8 \).

So far in our discussion, no dc magnetic field has been considered. For the quantum dot with an ac gate voltage and a dc magnetic field in the \( z \) direction, our results of the noise spectra for different voltage amplitudes are displayed in Fig.

FIG. 5. (Color online) Noise spectra with a rotating magnetic field in the \( xy \) plane with various amplitudes \( B_{ac} \). The peak position shifts with increasing \( B_{ac} \).
The dip position is fixed at the driving frequency.

When a nonzero dc magnetic field \( B_z \) is applied in the \( z \) direction, the frequency-dependent noise spectra of the quantum dot driven by a rotating magnetic field in the \( xy \) plane and a dc magnetic field in the \( z \) direction are presented in Fig. 8. The parameters used in the calculation are the same as in Fig. 5 and the dc magnetic field in \( B_z = 1.6 \). In Fig. 8, more peaks are observed than for vanishing dc magnetic field in Fig. 5. One can see that besides the peak determined by Eq. (51), there are both peaks (dips) fixed at \( \Omega \) and structures at positions depending on the ac-field amplitude \( B_{ac} \). All peak (dip) positions correspond to the differences of available Floquet quasienergies.

**IV. SUMMARY**

In this paper, the transport properties of a single-level quantum dot modulated by either an ac gate voltage or a rotating magnetic field have been studied within the Floquet quantum-master-equation approach in the sequential-tunneling limit. We have employed a generalized MacDonald formula to obtain the time-averaged current-noise spectra for both cases. Numerical results for the differential conductance and the frequency-dependent current noise have been presented. Besides the usual diamond structure due to the Coulomb blockade in the differential conductance, photon-assisted tunneling can give rise to additional lines parallel to the edges of the Coulomb diamond. These lines cannot survive inside the Coulomb diamond in the case of a rotating magnetic field. This is due to the fact that the rotating magnetic field only periodically rotates the spin direction while the energy of the electron on the dot remains unchanged. The frequency-dependent noise spectra of the quantum dot show additional peaks or dips in the presence of an ac field. The
behavior of these additional structures depends on the nature of the ac-driving field. In the case of an ac gate voltage, the position of the finite-frequency peak is fixed at the external ac frequency, independently of the voltage amplitude. On the other hand, in the case of a rotating magnetic field, the peak at nonzero frequency moves with changing amplitude of the rotating magnetic field. An additional dc magnetic field in the plane of the rotating magnetic field can also drastically change the noise spectra; it leads to the appearance of both movable and fixed peak structures in the noise spectra. All these peak positions are found to be determined by the energy differences between two Floquet transport channels.

**APPENDIX: DERIVATION OF THE GENERALIZED MACDONALD FORMULA WITH ac FIELD**

A derivation of the MacDonald formula for time-independent transport has been presented in Ref. 24. We show here that the periodicity of our time-dependent Hamiltonian makes it possible to estimate the noise spectra by generalizing the MacDonald formula. The derivation of the generalized MacDonald formula for the time-averaged noise spectra is outlined in the following.

We start from the Fourier-transformed current correlation function,

\[ S(t, \omega) = \int_{-\infty}^{\infty} dt' e^{i\omega t'} \langle \delta I(t), \delta I(t - \tau) \rangle, \quad (A1) \]

where \( \delta I(t) = I(t) - \langle I(t) \rangle \) and \([A, B] = AB + BA\). For simplicity, we omit the lead index in the noise expressions in this appendix.

From the definition of the current, we have

\[ \int_{t}^{t+\tau} dt' \delta I(t') = \int_{t}^{t+\tau} dt'[I(t') - \langle I(t') \rangle] = eN(t + \tau, t) - \int_{t}^{t+\tau} dt' \langle I(t') \rangle, \quad (A2) \]

where \( N(t + \tau, t) \) denotes the number of charges transferred during the interval from \( t \) to \( t + \tau \). Taking the expectation value of the square of this equation, we obtain

\[ 2e^2 \left( N(t + \tau, t) - \int_{t}^{t+\tau} dt' \langle I(t') \rangle / e \right)^2 \]

\[ = \left( \int_{t}^{t+\tau} dt' \int_{t}^{t+\tau} dt'' \langle \delta I(t') \delta I(t'') \delta I(t') \delta I(t'') \rangle \right) \]

\[ = \int_{t}^{t+\tau} dt' \int_{t}^{t+\tau} dt'' \int_{-\infty}^{\infty} d\omega \frac{1}{2\pi} S'(t', \omega)e^{i\omega(t'-t'')} \quad (A3) \]

Inserting the Fourier decomposition of the time-dependent relation

\[ S'(t', \omega) = S_0(\omega) + \sum_{k \neq 0} e^{-ik\Omega t'} S_k(\omega) \quad (A4) \]

into Eq. (A3), we obtain

\[ \ldots = \int_{t}^{t+\tau} dt' \int_{-\infty}^{\infty} d\omega \frac{1}{2\pi} S_0(\omega) + \sum_{k} e^{-ik\Omega t'} S_k(\omega) e^{-i\omega(\tau)} = \int_{-\infty}^{\infty} d\omega \frac{1}{2\pi} S_0(\omega) \frac{1}{\omega^2} (e^{-i\omega \tau} - 1)(e^{i\omega \tau} - 1) + \int_{-\infty}^{\infty} d\omega \sum_{k} \frac{1}{2\pi} S_k(\omega) \frac{1}{\omega(\omega + k\Omega)} e^{-ik\Omega t} (e^{-i(\omega + k\Omega)\tau} - 1)(e^{i\omega \tau} - 1), \]

\[ = \int_{-\infty}^{\infty} d\omega \frac{1}{\pi} S_0(\omega) \frac{1}{\omega}(1 - \cos(\omega \pi)) + \int_{-\infty}^{\infty} d\omega \sum_{k} \frac{1}{2\pi} S_k(\omega) \frac{1}{\omega(\omega + k\Omega)} [e^{-i\Omega t} - e^{-i(\omega + k\Omega)\tau} - e^{i\omega \tau} + 1], \quad (A5) \]

where we have used the notation \( \Sigma'_{k} = \sum_{k \neq 0} \). Differentiation with respect to \( \tau \) gives

\[ \frac{d}{d\tau} \left( 2e^2 \left( N(t + \tau, t) - \int_{t}^{t+\tau} dt' \langle I(t') \rangle / e \right)^2 \right) = \int_{-\infty}^{\infty} d\omega \frac{1}{\pi} S_0(\omega) \omega \sin \omega \tau + \int_{-\infty}^{\infty} d\omega \sum_{k} \frac{1}{2\pi} S_k(\omega) e^{-ik\Omega t} \frac{1}{\omega(\omega + k\Omega)} \left( -ik\Omega e^{-ik\Omega t} + i(\omega + k\Omega)e^{-i(\omega + k\Omega)\tau} - i\omega e^{i\omega \tau} \right). \quad (A6) \]

We next perform a Fourier transformation and take the time average over one period. Since the current correlation function is symmetric, \( S(t, t') = S(t', t) \), it can be shown that \( \tilde{S}(\omega) = \int_{0}^{\tau} dt' e^{i\omega t'} \rangle S(t, t') e^{i\omega(t'-t')} \rangle \) has the property \( \tilde{S}(\omega) = \tilde{S}(-\omega) \). We arrive at the generalized formula for the time-averaged noise spectrum for a periodic driving field,
\[
\frac{1}{T} \int_0^T dt e^2 \int_{-\infty}^{\infty} d\tau \frac{\partial^2}{\partial \tau^2} \left[ N(t+\tau,t) - \int_{\tau}^{\tau+\tau} dt' \langle I(t') \rangle / e \right]^2 \\
= \frac{1}{T} \int_0^T dt e^2 \int_{-\infty}^{\infty} d\tau \sin(\omega \tau) \frac{\partial^2}{\partial \tau^2} \left[ N(t+\tau,t) - \int_{\tau}^{\tau+\tau} dt' \langle I(t') \rangle / e \right]^2 \right)^2 = 2i \frac{\tilde{S}(\omega)}{\omega}.
\]

(A7)

Noting that the integrand of the \( \tau \) integral is even, we obtain the final result for the generalized MacDonald formula for the time-averaged noise spectrum,

\[
\frac{\tilde{S}(\omega)}{\omega} = 2e^2 \frac{1}{T} \int_0^T dt \int_{-\infty}^{\infty} d\tau \sin(\omega \tau) \frac{\partial^2}{\partial \tau^2} \left[ N(t+\tau,t) - \int_{\tau}^{\tau+\tau} dt' \langle I(t') \rangle / e \right]^2.
\]

(A8)

In comparison to the MacDonald formula for steady-state transport, an integration of \( t \) over one period is carried out to obtain the time-averaged noise spectra. The time average can also be expressed by an average over the initial phase of the ac field. Hence, our expression is equivalent to the form given by Clerk and Girvin. 46

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72 W. Shockley, J. Appl. Phys. 9, 635 (1938).